

Concrete Pictorial Abstract (CPA) Model as a Teaching Strategy in General Mathematics

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Abstract — The Concrete Pictorial Abstract (CPA) model as a teaching strategy in General Mathematics simply helps students visualize abstract problems and make them more accessible and to ensure that students develop a tangible understanding of the math concepts and skills. The respondents of the study were Grade 11 students under the HUMMS strands, Division of Pangasinan II. Documentary analysis was utilized to interpret the results in the diagnostic and summative tests in General Mathematics during the school year 2022-2023. The test consisted of 50 number of items classified based on concepts, process and metacognition. After a systematic and in-depth analysis of the data gathered, the findings are: (1) The students' level of performance in the diagnostic test is fair while a very good performance in the summative test; (2) The level of mathematical ability of Grade 11 students showed that there is an improvement in the summative test using the CPA model than in the diagnostic test as to concepts, process and metacognition. Hence, the students obtained a higher score. (3) There is a significant difference between the students' performance on the diagnostic and summative tests. Therefore, the students' assessment performance was improved after exposure to the CPA model but some students needs effort to improve on the area of metacognition in both diagnostic and summative test. Hence, a learning development plan as an intervention is highly recommended for adoption to improved students' mathematical abilities.

Keywords — **Concrete, Pictorial, Abstract, Model, Teaching Strategy**

I. Introduction

Mathematics teachers must actively participate in their students learning to effectively impart the rules, definitions, and processes that learners must master. Learners who are actively involved have a greater understanding of mathematical concepts. This aligns with the concrete, pictorial, abstract (CPA) methodology which allows for in-depth and sustainable comprehension of concepts in mathematics.

Mathematics education in the Philippines has been a concern for many years. The Philippines ranked second to last in science and mathematics and last in reading out of 79 participating nations, based on the findings of the 2018 PISA, the Programme for International Students Assessment. Filipino students were among the student groups with the lowest ratings for

performance. Less than 20% of students in mathematics achieved the required level of ability (Level 2), and more than 50% showed deficient performance (below Level 1). These Filipino pupils have fallen behind in mathematics education, scoring below the PISA lowest level of competency. More than half of this age group of Filipino students perform inadequately in mathematics compared to their classmates in other areas. Between students in public and private schools, where the mean was 343 and 395, there were substantial differences in the degree of low math performance, (Department of Education 2019). The results of the TIMSS in 2019, which examined Grade Four students' proficiency in math and scientific skills, did not show much improvement in the Philippines. It ranked the lowest among the 58 countries included in the study. According to (Mullis et. al 2019) the Philippines received a math score of 297 in the 2019 TIMSS result. Additionally, in the [2019 Southeast Asia Primary Learning Metrics \(SEA-PLM\)](#), the Philippines similarly performed poorly, which measured the capacity of Grade 5 students in reading, writing, and mathematics. The country performed below the regional average in all three areas. Numerous factors, such as a lack of trained teachers, a shortage of supplies and tools, and an out-of-date curriculum, have been attributed to this poor performance, (Puspitaningrum et al. 2021). According to (Benedicto, P. N., & Andrade, R. 2022), Filipino students excel in knowledge acquisition but suffer with critical thinking skills, which hinder their achievement in mathematics. In addition, the government stresses that there is a need to improve the standards of education, particularly in both science and mathematics, and that instruction for students' needs to be strengthened.

Before the pandemic, the Philippines mathematics education faced challenges and consistently scored poorly on international assessments, (San Juan, 2019). However, Philippine education has been more challenging during the pandemic, particularly in teaching and learning mathematics. Modular distance learning is a challenge for both teachers and students, particularly when it comes to teaching and learning mathematics subjects, because specific knowledge and skills are required. Additionally, (Contini et al. 2021) found that student performance in mathematics decreased during the pandemic. The COVID-19 pandemic is more likely to result in a significant decline in the passing rate of high school students in National Examinations (Sintema, 2020). Educators, policymakers and stakeholders must recognize these challenges and work together to address them effectively.

Moreover, the mean percentage score in Mathematics on the National Achievement Test dropped below the (DepEd, 2019) standards. The cornerstone of knowledge in science and technology is mathematics, precisely dynamic in a country's economic development. Therefore, to improve mathematics education in the Philippines, it is imperative to address the issue of students need for more confidence in the subject.

Furthermore, the Department of Education conducted a Diagnostic Test last October 20-21, 2022; the teachers teaching General Mathematics in the division of Pangasinan II, in the province of Pangasinan assigned to send reports on the least mastered competencies in the said

subject. It was found that the least mastered competencies are: solves problems involving logarithmic functions, equations and inequalities; finds the value and present value of both simple annuities and general annuities; perform addition, subtraction, multiplication, division, and composition of functions; finds the domain and range of an inverse function; and solve problems involving rational functions, equations, and inequalities.

In response to this problem, one effort that can be made is the introduction of the Cognitive, Pictorial, Abstract (CPA) approach to mathematics instructions, which is being explored as a potential solution to the low-achieving performance of students in Mathematics education in the Philippines. Instead of mindless memorizing facts and formulas, this approach focuses on problem-solving and critical thinking abilities. This method promotes a more active and engaged by allowing students an opportunity to demonstrate their knowledge and skills to challenges in the real world.

Literature Review

There were studies that would relate to the present study. This study explains further the effectiveness of the Concrete-Pictorial-Abstract (CPA) model as a teaching strategy in General Mathematics.

Incorporating visual and concrete representations in mathematics instruction influences student's awareness and regulation of their learning process (Mokhtar et al. 2019). Using CPA promotes metacognitive skills, facilitating students understanding of abstract mathematical concepts and enabling them to monitor their learning and apply appropriate problem-solving strategies. It also supported by the study of (Salimi et. al 2020), who strong evidence supporting the effectiveness of the CPA approach in enhancing mathematical learning. Additionally, The Concrete-Pictorial-Abstract technique that improves students' mathematical representation abilities by allowing them to develop information through physical and mental reflection, hence increasing equation-solving abilities (Azmidar, A., & Husan, H. 2022). Similarly, (Liu et al. 2019) found that incorporating concrete, pictorial, and abstract representations in mathematics instruction enhance students learning experiences and promote a deeper understanding of mathematical concepts.

Additional research by (Wong et al. 2020) found that the CPA technique helps students who struggle with learning because it gives them tangible manipulatives and visual aids to help them comprehend abstract mathematical ideas. This is also supported by the study of (Lim Y. and Zhou Q. 2019), which investigates the effectiveness of a concrete-to-abstract method in teaching mathematics to students in inclusive classes who have difficulties with learning. The study found that highlighting its efficacy as an instructional strategy to improve the mathematical performance of diverse student populations. Further, (Cuison, L. 2020) investigated the effect of the concrete, pictorial, and abstract instructional approach on the mathematics achievement and mathematics anxiety of Grade 7 students. The study results indicated that students who received instruction

using the CPA model showed significantly higher gains in mathematics achievement than the other group.

Furthermore, (Tournaki et al. 2020) discussed how the CPA model provides multiple entry points into mathematical concepts, catering to the diverse needs of the students by starting with concrete manipulatives, progressing to pictorial representation, and moving to abstract symbols where teachers can scaffold instruction and support students understanding at various levels.

Several studies discovered that the CPA model of Singaporean math instruction was highly efficient. One is the study of (Jackson's 2020), the study discovered that students who got an education using the Singapore Math technique outperformed those who received instruction using other methods. The findings imply that the Singapore Math approach can be an effective teaching tool for boosting student's mathematics success in various educational contexts. According to (Alvin, B. 2020), the Concrete, Pictorial, Abstract (CPA) method of the Singapore Math model has been proved to be beneficial in improving mathematics learning. It has been discovered that the model method—which is backed by a constructivist learning design—is superior to conventional lecture and practice techniques in terms of helping students grasp mathematical topics at a deeper level. In addition, according to research by (Dumdum et al. 2022) students' performance in problem-solving is greatly enhanced when the Singapore Math Model Method is applied, the study also found out that Math performance is greatly enhanced by the Singapore Math approach. In comparison to conventional teaching approaches, it is more effective. Furthermore, a study by (Zulfakri et.al, 2019) demonstrated that in comparison to the conventional learning group, students who were instructed using the CPA technique improved more in representation and mathematical problem-solving skills.

This is also supported by the study of (Duan, 2019), who found that students who received instruction using the Singapore Math model outperformed those who received instruction using traditional methods in algebraic equations. Further, The model method is an effective alternative for conventional teaching techniques because of its emphasis on problem-solving, concrete-pictorial-abstract approach, and schematic representations, which help to improve algebraic thinking and problem-solving abilities (Tan et. al, 2022). This implies that the Singapore Math approach may help boost student's mathematics achievement in various nations and educational environments.

In addition, the achievement and enhancement of the students mathematical reasoning ability by learning with the CPA approach were better than those with conventional learning, (Putri et al. 2020). CPA can improve the mathematical representation (Nugroho and Jailani 2019); student learning outcomes (Yuliyanto, Putri et al. 2019), increase spatial sense (Putri, 2019), students' self-efficacy (Yuliyanto, Turmudi, et al.2019), elementary school students' self-efficacy in mathematics (Putri et al.2020), and reduce mathematical anxiety (Putri et al.2019). These studies demonstrate the importance of effective mathematics education and highlight the potential benefits of adopting approaches like the CPA approach to improve Mathematics performance.

It is important to highlight that this review of literature provide additional knowledge in the conduct of the current study.

II. Methodology

Research Design

The descriptive documentary analysis approach was used in the study. The research used descriptive documentary analysis in quantitative counts using diagnostic and summative tests in General Mathematics of HUMSS students of the Division of Pangasinan II for the school year 2022-2023.

The respondents of the study were the Grade 11 HUMSS students of the Division of Pangasinan II during the school year 2022-2023. The respondents are from the different senior high schools with a total of 665 respondents.

The instrument used was the diagnostic and summative tests in General Mathematics during the school year 2022-2023. It was composed of 50 number of items classified based on concepts, process and metacognition.

III. Results and Discussion

This chapter present, interprets and analyzes the data of the study.

It was shown in the table in the next page that the students' performance level in their diagnostic test has a percentage of 59.40 with a frequency of 395, which is equivalent to 'Fair'. This shows that the student's performance in their diagnostic tests must be improved. Students may experience test anxiety during diagnostic tests, which impairs their ability to recall information and perform to the best of their abilities. Time constraints in tests can increase student's anxiety, negatively affecting their mathematical ability (Mendoza et.al 2019). And the students' performance level in their summative test has a percentage of 41.20 with a frequency of 274, equivalent to "very good." This means the students' performance level increases when the CPA is implemented.

Table 1. Level of Performance of the Students in Diagnostic and Summative Tests

Score Interval	Diagnostic Tests			Summative Tests		
	Frequency	Percentage	Level of Performance	Frequency	Percentage	Level of Performance
41-50	25	3.76	Outstanding	105	15.79	Outstanding
31-40	45	6.77	Very Good	274	41.20	Very Good
21-30	173	26.02	Average	252	37.89	Average
11-20	395	59.40	Fair	33	4.96	Fair
0-10	27	4.06	Poor	1	0.15	Poor

Also, the impact of instructional strategy on the comprehension of mathematical concepts by students and their ability to apply problem-solving skills may affect their performance in mathematics. This is also supported by the study of (Borji et al. 2019), conceptual instruction improves mathematical test performance over time when compared to procedural teaching. This study provides evidence supporting the effectiveness of the CPA model in helping students develop a deeper understanding of general mathematics. Therefore, student's assessment performance tends to improve after exposure to the CPA model. They demonstrate higher achievement levels and more proficient problem-solving skills, leading to better outcomes on tests and examinations.

Table 2.1 Level of Mathematical Ability of students in CPA along Concepts

	Minimum	Maximum	Mean	Median	Std. Deviation	Skewness	Kurtosis
Diagnostic	11	31	17.43	16.18	5.43	0.912	0.236
Summative	21	33	26.55	25.91	3.21	0.448	-0.692

Table 2.1 shows the students mathematical ability along concepts. The minimum score of the students along concept in their diagnostic test is 11, while the maximum score is 31 with a mean of 17.43, median of 16.18 and a standard deviation of 5.43. Their low scores suggest that the students struggle to grasp fundamental concepts and apply them to solve problems requiring deeper understanding. According to (Velez et.al 2023) found that students struggle with understanding mathematical concepts and efficiently manipulating geometric forms and spaces. Mathematical representations play an important role in improving the understanding of mathematical concepts and interpreting and solving problems.

Moreover, the distribution is slightly skewed to the right, indicating relatively fewer low scores and some higher scores. Additionally, the distribution is less peaked than average, suggesting that the scores are spread out with lighter tails.

The students performed relatively well on the summative test with a minimum score of 21 and maximum score of 33 with a mean of 26.55 and median of 25.91 and standard deviation of 3.21. The student's performance in the summative tests along concepts is relatively good on

average, with moderately variability in scores, a slight skew to the right. A study by (Liu et al. 2019) found that incorporating concrete, pictorial, and abstract representations in mathematics instruction enhance students' learning experiences and promotes a deeper understanding of mathematical concepts. Additionally, the CPA technique is especially beneficial for children who struggle with learning because it offers them tangible manipulatives and visual aids that enhance their comprehension of abstract mathematical ideas, according to research by (Wong et al. 2020). Therefore, students low scores in mathematical concepts in General Mathematics subjects could stem from various factors related to students understanding, skills, motivation, and external influences. Identifying the root causes of low performance is crucial for implementing targeted interventions and support strategies to help students improve their mathematical learning outcomes.

The implementation of CPA typically leads to higher levels of mathematical ability along concepts due to its emphasis on building a solid conceptual understanding of mathematical concepts. This result can be supported by the study of (Lim Y. and Zhou Q. 2019), investigates the effectiveness of the concrete-to-abstract approach in teaching mathematics to students with learning difficulties in inclusive classrooms. The study found that highlighting its efficacy as an instructional strategy to improve the mathematical performance of diverse student populations.

With this, using concrete pictorial abstracts likely facilitated students' comprehension of mathematical concepts at a deeper level. Abstract ideas are more tangible and accessible, helping students grasp foundational principles more effectively.

Table 2.2 Level of Mathematical Ability of students in CPA along Process

	Minimum	Maximum	Mean	Median	Std. Deviation	Skewness	Kurtosis
Diagnostic	9	19	13.36	13.29	2.75	0.694	0.349
Summative	19	30	25.09	25.47	3.25	-0.574	-0.287

As can be gleaned from the table above, it can be seen that the minimum average score of the students in their diagnostic test is 9, and the maximum score average score is 19, with an average mean score of 13.36, median of 13.29 and a standard deviation of 2.75. This indicates that the scores of the students in their diagnostic tests under process is fairly homogeneous. It also indicates that the scores of the students in their diagnostic tests are somewhat lower than the mean scores along concepts. This could mean that the students are better at understanding than processing concepts. Moreover, the students got higher mean scores in their summative tests compared to their diagnostic tests, with an average mean score of 25.09 and a median of 25.47 and a standard deviation of 3.25. The student's performance in the summative test reflects an average score with moderate variability, a moderate skew to the left and a moderate flattened distribution, underscoring varying degrees of comprehension and proficiency among the student cohort.

General Mathematics courses often prioritize procedural knowledge and problem-solving skills over conceptual understanding. Based on research conducted by (Atoyebi, O. M., & Atoyebi, S. B. 2022), Mathematics anxiety has a negative impact on students' mathematical thinking, particularly their problem-solving and reasoning ability. That is why students may focus more on memorizing the algorithms and procedures to solve mathematical problems rather than developing a deep understanding of underlying mathematical concepts.

A study by (Azmidar, A., & Husan, H. 2022) explored the Concrete-Pictorial-Abstract technique that improves students' mathematical representation abilities by allowing them to develop information through physical and mental reflection, hence increasing equation-solving abilities. Their findings indicated that students exposed to concrete and pictorial representation before tackling abstract symbolic representations performed better and exhibited deeper conceptual understanding than those who only encountered abstract representations.

Therefore, positive outcomes associated with using concrete pictorial abstracts suggest that it is an effective instructional strategy for promoting mathematical processes in General Mathematics. This approach aligns with research-based practices emphasizing active learning and visual representations to support student learning.

Table 2.3 Level of Mathematical Ability of students in CPA along Metacognition

	Minimum	Maximum	Mean	Median	Std. Deviation	Skewness	Kurtosis
Diagnostic	11	17	12.72	12.47	1.73	1.768	3.546
Summative	19	29	24.33	25.29	3.59	-0.471	-0.984

Metacognition is a mental process where students think about their thinking. The student's ability to regulate their thinking process is crucial in mathematical learning and problem-solving. Relating to the table above regarding the level of mathematical ability of the students in CPA along with metacognition, it is shown that the minimum average score of the students is 11, and the maximum average score is 17 with a mean of 12.72, median of 12.47 and the standard deviation of 1.73. This means that the mean scores of the students are concentrated within the mean and that the student's level of mathematical ability and metacognition is high. This indicates that these high-ability students with metacognition engage in strategic learning approaches where they can monitor, set goals, evaluate their progress, and enhance their learning efficiency and effectiveness.

Meanwhile, the average scores of the students on their summative test after the implementation of the concrete, pictorial, and abstract models are higher compared to the diagnostic test. Students with high mathematical ability and metacognition think critically about their problem-solving processes. This means students can analyze their strategies, consider alternative approaches, and even evaluate their progress in mathematics.

The positive skewness of 1.768 is notable, indicating a pronounced skew to the right in the distribution of scores. This skew suggests that there may be relatively fewer lower scores and a

concentration of higher scores among certain students, possibly indicating a subset of individuals who possess more vital metacognitive skills, such as self-awareness and strategic learning approaches. Moreover, the high kurtosis value of 3.546 signifies a substantial level of peakedness in the distribution, suggesting that the scores are heavily concentrated around the mean with relatively few outliers. In summary, the students' performance in the summative test of General Mathematics, when examined through the prism of metacognition reflects an average score with moderate variability, a pronounced skew towards higher scores and a distribution that is heavily concentrated around the mean, underscoring the potential influence of metacognitive abilities on academic achievement in mathematics.

The outcomes of the summative test in General Mathematics unveil valuable insights into the student's mathematical prowess, which can be viewed through the lens of metacognition. With a mean score of 24.33, it is evident that the students, on average, achieved a score close to this mark. The standard deviation of 3.59 implies a moderate degree of variability in scores around the mean, indicating that while some students performed around the average, others displayed more significant deviations in their understanding and application of mathematical concepts. The negative skewness of -0.471 suggests a slight skew to the left in the distribution of scores. This skew implies that there may be relatively fewer higher scores and a concentration of lower scores among certain students, potentially indicating areas where metacognitive strategies could be further developed to enhance performance. Furthermore, the negative kurtosis of -0.984 indicates a flattening compared to a normal distribution, suggesting that the scores are somewhat dispersed, with a broader range of performance levels across the student cohort. In summary, when viewed through the prism of metacognition, the student's performance in the summative test of General Mathematics reveals an average score with moderate variability, a slight skew to the left, and a distribution that is somewhat flattened, highlighting potential opportunities for the cultivation of metacognitive skills to bolster mathematical achievement.

A study by (Mokhtar et al. 2019) explores how incorporating visual and concrete representations in mathematics instruction influences students' awareness and regulation of their learning process. The study's findings suggest that using CPA promotes metacognitive skills, facilitating students' understanding of abstract mathematical concepts and enabling them to monitor their learning and apply appropriate problem-solving strategies.

The researcher concludes that students exhibit strong metacognitive regulation skills, contributing to their success in general mathematics. Using CPA methods will likely result in positive learning outcomes in general mathematics. Overall, this result indicated that implementing the CPA sequence significantly improved students' mathematical achievement and conceptual understanding compared to traditional instruction methods. Mathematical ability along concepts, process, and metacognition suggests that students are not just memorizing information temporarily but are internalizing mathematical concepts and problem-solving strategies for long-term

retention. Concrete representations aid in forming robust mental schemas, enhancing the student's ability to recall and apply mathematical knowledge over time.

Table 3. Percentage of the Mathematical Ability of Grade 11 students along Concepts, Process and Metacognition in Diagnostic and Summative Test

	Concepts		Process		Metacognition	
	Diagnostic	Summative	Diagnostic	Summative	Diagnostic	Summative
81-100		13.6%				
61-80	12.5%	63.6%		75.0%		25.0%
41-60	37.5%	22.7%	16.7%	25.0%	10%	75.0%
21-40	50%		83.3%		90%	
1-40						

The percentages of the mathematical ability of Grade 11 students, along with concepts, processes, and metacognition in their diagnostic and summative tests, are shown in the table above.

The table shows higher percentages on the summative test than on the diagnostic test of the Grade 11 students in all areas of their mathematical ability. This means that the CPA model allows for the differentiation of instruction based on students' varying learning styles and abilities. The CPA provides multiple entry points into the material, where teachers can address the diverse needs of their students, leading to improved performance. A study by (Tournaki et al. 2020), discussed how the CPA model provides multiple entry points into mathematical concepts, catering to the diverse needs of the students by starting with concrete manipulatives, progressing to pictorial representation, and moving to abstract symbols where teachers can scaffold instruction and support students understanding at various levels.

Therefore, the CPA model's effectiveness in promoting higher gains in mathematics achievement can be attributed to its emphasis on building conceptual understanding, providing visual representations that help them improve their mathematical processes, and supporting knowledge retention and transfer.

As shown in the table in the next page, there is a significant difference between the performance of the students in their diagnostic test and summative test, with a t-value of -30.09 and a significance level of .002. This means that the observed change in performance is likely to occur because of the utilization of the CPA model. The null hypothesis, which reads; "There is no significant difference between the performance of the Grade 11 students before and after the utilization of the CPA model," is implied to be rejected by this evidence. The CPA model has had a measurable impact on student performance that could manifest as improvements in understanding concepts, problem-solving abilities, and improvements in test scores.

Table 4. The Difference in the performance of Grade 11 students before and after the utilization of the CPA model

	Mean	Mean Difference	t	Sig.
Diagnostic	20.2150	-12.21	-30.09	.002
Summative	32.4226			

A study by (Cuison, L. 2020) investigated the effect of the concrete, pictorial, and abstract instructional approach to mathematics achievement and mathematics anxiety of Grade 7 students. The study results indicated that students who received instruction using the CPA model showed significantly higher gains in mathematics achievement than the other group.

Concrete manipulation in the CPA model promotes hands-on, experiential learning, which can increase student's engagement and motivation. Students who can manipulate physical objects to explore mathematical concepts are more likely to develop a deeper understanding.

IV. Conclusion

Based on the study's findings, the following conclusions were made: (1) Students assessment performance improved after exposure to the CPA model. It demonstrates higher achievement levels and more proficient problem-solving skills, leading to better outcomes on tests and examinations. (2) Students are needing improvement on metacognition in both diagnostic and summative test. (3) The used of CPA models improved the mathematical ability of Grade 11 students along the three areas namely: concepts, process and metacognition. (4) There is a significant difference between the performance of the Grade 11 students along concepts, process and metacognition. From the result of the findings and conclusion that were made, the researchers offer the following recommendations: (1) The mathematics teachers shall implement the CPA model in Mathematics instruction since it effectively enhances the mathematical abilities of the students. (2) Provide opportunities for teachers through seminars, workshops, or peer collaboration sessions utilizing the CPA model that focuses on metacognition. (3) Adopt the proposed learning development plan to improve the student's mathematical ability and promote a deeper understanding of mathematical concepts, processes and metacognition. (4) Further qualitative investigation be conducted to explore students' experiences and perceptions of instructional interventions like the concrete, pictorial and abstract models in Mathematics.

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